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Study and validation of a fast ray tracing method to determine density flux for symmetric solar optical systems

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ABSTRACT

In this research paper, a computational program called MCMRDF-3D, is developed and validated by comparison with a reference software. MCMRDF-3D is formulated on Multi-Ray Monte Carlo Method for the determination of the distribution of solar irradiation along the receiver of any solar-concentrating system that withstand translational or rotational symmetries. This code incorporate an advantageous approach that consist in estimating the overall distribution of irradiation of a large scale solar concentrating systems on the basis of a consistent sample, then saving computation time, processor capacity and storage memory. The paper presents the new approach and applied it to determine the distribution of irradiation provided by three concentrators. We compare the processing time, the irradiation map rendering, and the size of numeric data, to those provided by a Reference software. The obtained irradiation distribution and power harvesting quantities over the receiver matches very well. We conclude that MCMRDF-3D proves to easy, fast, accurate and reliable approach for the simulation of solar concentrator system. Among other implications, this work opens a path for saving time when studying numerous symmetrical imaged and non-imaged optical systems. For future work, this time saving and flexible program will be of a significant help to envision optical errors from real behavior of optical surface in concentrating systems.

RESUME

Cet article de recherches, présente une procédure de calcul des transferts radiatifs implémentée dans un programme informatique que nous avons dénommé MCMRDF-3D, et les résultats des tests de performance en comparaison avec un logiciel de référence dans le but de sa validation. L'approche générale MCMRDF-3D repose sur la méthode Monte Carlo du lancé Multi-Rayon de détermination de la distribution de l'irradiation solaire le long du récepteur de n'importe quel système à concentration. La particularité de notre approche réside dans une application simplifiée pour l'étude de problèmes qui présentent des symétries de translation. La variante de calcul que nous avons implémenté dans ce programme exploite une approche avantageuse qui simplifie l'estimation globale de la distribution de l'irradiation des systèmes de concentration solaires à une opération de transposition mathématiques d'un échantillonnage cohérent. Ce qui entraîne des économies en termes de temps de calcul, de capacité de processeur et de mémoire stockage. Nous présentons les résultats d'application de la nouvelle approche pour l'analyse de la distribution d'irradiation fournies par trois concentrateurs pris en exemple. Pour ces exemples, nous comparons les temps de calcul, les profils de distribution d'irradiation, et la taille des données numériques, aux résultats d'un logiciel pris en référence. Les distributions d'irradiation et les puissances obtenues sont globalement identiques avec des écarts non significatifs. Nous en concluons que MCMRDF-3D est une approche de calcul des transferts radiatifs précise, fiable et bien plus rapide. Entre d'autres implications, ce travail offre des perspectives d'économie de temps pour l'analyse des systèmes optiques symétriques et permet d'envisager la prise en compte d'erreurs optiques associées au comportement réel des surface optique.

I. INTRODUCTION

One of the most globally approaches used to perform a statistically viable analysis of an optical system is radiative transfer modeling (Baud G. et al., 2013. and Myriam D. et al., 2013.) On one hand, it permits the estimation of the quantity of energy received by the receiver, and on the other hand, the determination of the spatial distribution of this energy (Blanco M.J., 2003.).

Veynandt F., 2011. and Farges O. et al., 2012. in there works, modelled a solar energy thermal conversion process throughout an integral formulation of the Monte Carlo method, with the aim of providing the flow map, at the height of the receiver and optimizing the geometry by varying parameters. The works done by Veynandt F., 2011., Farges O. et al., 2012., and De la Torre J.D., 2011. highlighted three advantages of the Monte Carlo method. As a statistical method applied to the calculation of particle transport, it makes it possible to simulate the transport of photons in the most complex geometries, a fortiori those of multi-reflective optical systems; without the need to resort to experimental results.

For applications in radiation, radiative transfers are considered as linear transport phenomena where the particles are photons that do not interact with each other either directly or indirectly. Many methods adopt the hypothesis that incident irradiation on an optical system follows the path of the ray. This concept is considered as a sufficient approach to address the physics of radiative

transfers. This approach is shown to be suitable for many cases of study but limits the domain of investigation particularly when peculiar behavior of light interaction with matter is admitted.
Generally, the modeling of the interactions between the incident solar radiation and the real surfaces of the solar concentrating system is addressed within the domain of geometrical optics, i.e., neglecting the finiteness of the wave length (Wendelin T., et al. 2013.). So physical optics behaviors and statistical physics behaviors are avoided when considering the interaction of light with matter. Until now, the calculation process is huge with a considerable number of rays to be considered at the beginning and the number of interactions and behavior undertaken by each ray all over it path. This is self-explanatory when all along the path of each ray one has to consider irregular repartition of light after a given interaction that appends often in real case with non-specular reflection, irregular splitting or absorption of polychromatic light. Most of the time the convolution is the approach at least considered to deal with non-uniform redirection of light.
In this paper we develop an approach of calculation implemented in an algorithm called Méthode de Monté Carlo à Multiples Rayons pour la Determination du Flux en 3 Dimensions (MCMRDF-3D) (Stanislas Sanfo., 2015) which reduces the number of ray's realization at the beginning in order to reduce the weight of numerical operations. Such an approach opens the path for considering both physical optics and statistical physics behaviors over the path of light. We present next the MCMRDF-3D approach and compare its results with those of the used reference software.

II. METHODOLOGY

II.1. Nombre de pages

MCMRDF-3D is an algorithm implemented on Matlab. This optical modeling code is globally based on ray tracing inspired from Monte Carlo method. The mathematical formulation presented by Veynandt F., 2011., De la Torre J.D., 2011., and Carlini M. et al., 2011. is based on the Monte Carlo method, while the algorithmic form as perceived by Veynandt F., 2011. is inspired by ray tracing procedures. For it description, we consider the problem of determining the map flow of irradiation for a two-stage concentrating system as illustrated by Fig.1. The variables, x_0, x'_0, x_1, x'_1, x_2, and x_3 are associated with the location of the interaction points of rays beam at different locations which are the aperture of the collector and the following three components: the collector, the secondary reflector and the receiver.
From both the works done by Farges O. et al., 2012. And De la Torre J.D., 2011. we can express the integral for the calculation of the surface flux as follows:

$$\psi(R \in dS_R) = \int_{S_{rp}^{+}} \int_{\Omega_S} K \times P_{X_1}(x_1) P_{\Phi}(\omega_S) dx_1 d\omega_S \quad (1)$$

Where K is a function which carries the criteria of ray interactions over its path from sun to the receiver, throughout the two reflection stage, with oriented optical surface components stated by +/- as indicated in **Erreur ! Source du renvoi introuvable.**. It is the probalistic function for a given ray to reach a given point R after some effective interactions with reflectors:

$$K = H(R|\omega_S \cap x_1) \times ὢ_{,2} \quad (2)$$

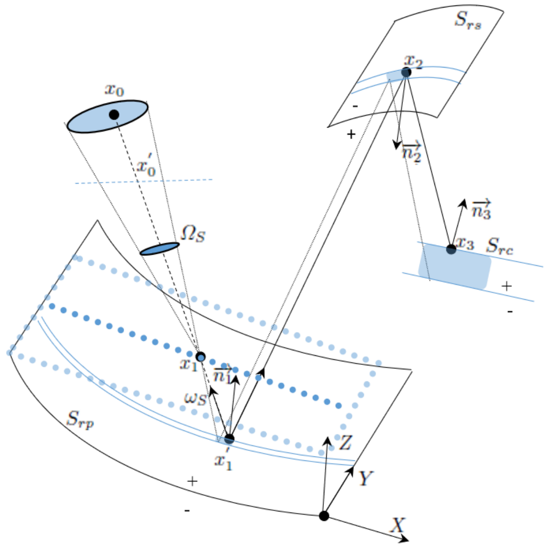

Fig.1. *Illustration of the new approach based on linear symmetric solar concentrator*

Eq.1 involves two main integrals. The first integral is applied on space along the curvature of the concentrator. Here we make the choice of the aperture of the collector in order to consider an uniforming density of rays beam at that height. For each position occupied at the collector aperture, we apply a second integral of solid angle in the 3D field of view of the sun. It is assumed that all reflections are specular and therefore there is only one direction for each reflected ray beam. The second integral is applied over the entire angular field of view of the sun.
For photons subjected to shading or blocking phenomena $H(R|\omega_S \cap x_1) = 0$, while for the effective impact of photons on the receiver at a point R covered by the element of surface dS_R $H(R|\omega_S \cap x_1) = 1$.
The evaluation requires some probabilistic density functions (*pdfs*). According to De la Torre J.D., 2011. when the *pdfs* are well chosen according to the problem, they make it possible to improve the convergence of the algorithm while reducing the variance of each variable. The *pdfs* are stated as follow:

$$p_{X_1}(x_1) = \frac{1}{S_{rp}} \quad (3)$$

$$p_{\Omega_S}(\omega_S) = \frac{1}{2\pi(1-cos\omega_S)} \quad (4)$$

When a ray impacts the receiver inside an elementary area dS_R established at the point R, that ray contributes to the flux in that specific area. This contribution takes the form of a Monte Carlo weight which, in our case, is given by Eq.5:

$$ѽ_{,2} = \frac{I_S.(\omega_S.n_h)}{p_{\Omega_S}(\omega_S)\times p_{X_1}(x_1)} \quad (5)$$

Thanks to the irradiation profile, the total power over the receiver is determined by integration along the surface of the receiver:

$$P = \int_{R\in S_{rc}^{+}} \psi(P) dS_R \quad (6)$$

Determining the optical path of a photon involves locating its interaction points. An interaction that follows a reflection led to a transformation aimed at changing the direction and meaning of incident vector. The two specular reflections are stated as follows:

$$\overrightarrow{u_{R_1}} = \overrightarrow{u_I} - 2(\overrightarrow{n_{RP}}.\overrightarrow{u_I})\overrightarrow{n_{RP}} \quad (7)$$

$$\overrightarrow{u_{R_2}} = \overrightarrow{u_{R_1}} - 2(\overrightarrow{n_{RS}}.\overrightarrow{u_{R_1}})\overrightarrow{n_{RS}} \quad (8)$$

Where

$\overrightarrow{u_{...}}$ are units vectors carrying the direction of rays beams

$\overrightarrow{n_{...}}$ represents the normal to an optical surface at a giving interaction point

RP and RS are indices which refer to primary reflector and secondary reflector, respectively

I, R_1 & R_2 are indices which indicate the origin of the photon.

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II.2. Description of the new approach

Generally, solar radiation is assumed to consist of discrete beams of energy, called photons, so that the algorithmic formulation of the integral calculus formulated by Eq.1, boils down to following the optical paths of the photons, from the source of the rays to the receiver. We still considered this in our new approach which consists in modeling the problem assuming that generally the problems assume a global symmetry or a local symmetry. Global symmetry can be seen for the case of V-trough parabolic concentrator for instance where there is a perfect symmetry in the longitudinal direction. Local symmetry can be seen over each mirror used for giant concentrating Tour. For this last case one can seek for local symmetry for each mirrors. On this basis we operate through the following steps as presented in **Erreur ! Source du renvoi introuvable.**:

- (1) We look for symmetric configuration of the problem: global symmetry or local symmetry;
- (2) we realize a uniform discretization over non-symmetric plane at the collector aperture in $N_x \times N_y$ pixels $P_{ij}/(i,j) \in \{1,\cdots,N_x\} \times \{1,\cdots,N_y\}$ of equal size $\delta_x \times \delta_y$;
- (3) we consider the same number N_r of rays coming from the cone of view of the sun and the same distribution of these rays, for each given point P_{ij};
- (4) we subdivided the surface of the receiver into identical small square area of equal size $\delta_x \times \delta_y$ and limited by points $R_{kl}/(k,l) \in \{1,\cdots,M_x\} \times \{1,\cdots,M_y\}$.

We assume that the properties of solar radiation are identical at all points P_{ij}. According to Veynandt F., 2011. it is interesting to see concentration as a multiplication of the sun. It is not a superposition, but a juxtaposition of images of the sun obtained by using a converging lens or a reflective surface of peculiar shape. Considering this principle, we divide our simulation approach of the physical problem of particle transport into two steps:

- In the first step, we draw a sample of irradiation distribution. We start by considering the plane that undergoes this symmetry. The intersection line between this plane and the aperture contains N_x points P_{ij}, $i \in \{1 \dots N_x\}$, where j states the location of the plane. The calculation of rays paths from the sun to the points P_{i1} until the receiver, led to a sample of irradiation distribution maps on pixels of dimensions $\delta_x \times \delta_y$ over the receiver (refer to Fig.1). Each pixel, is associated to the amount of energy that it intercepts.
- In the second step, we determine the flow map of energy over the receiver on the basis of the sample. In order to do that, we integrate the contributions of remaining elementary surfaces located at points P_{ij} , $i \in \{1,\dots,N_x\}$ and $j \neq 1$. This operation resumes to transposing the previous sample by a step δ_y, along the symmetry axis of translation.

II.3. Description of reference software

The reference software we used is TONATIUH software. It is a ray tracing software considered in several writings as the reference software for design and simulation of the energy behavior of solar concentrating systems (Wendelin T., et al. 2013., and Sebastian-James et al., 2012.). This program is based on the Monte Carlo method. It offers a graphical interface on which it is possible to visualize the modeled concentrator and the trajectory of the rays. However, the results of a simulation must be exported as a binary files which need to be post-process using a mathematical calculation software. The most dedicated environment for this purpose showed to be the Mathematica software, in 2014. It will also be the one adopted in this study.

The outline of the simulation process with Reference are:

- (1) hub modeling;
- (2) choosing a model of the sun;
- (3) a model of the interactions between radiation and concentrator elements; and
- (4) specification of the results the program should produce.

The modeling of a reflector is carried out by identifying its geometry in the database, and by defining its aperture, its extent, its geographical position, and the optical properties of its two faces. This software package will serve as a basis for the comparison of the proposed approach.

II.4. Concentrators setup cases for the comparative study

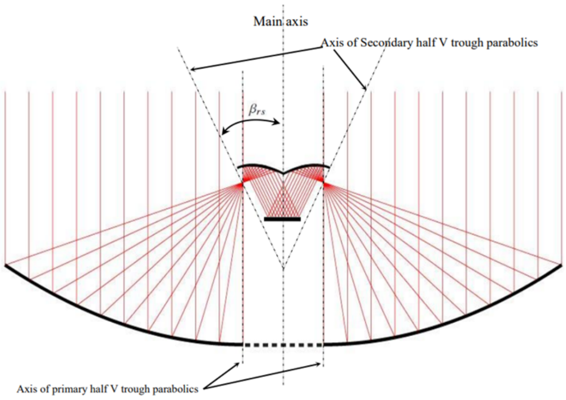


Fig.2. *Configuration of the two stage concentration systems considered as study cases*

The study cases are linear concentrators of 0.5 m length made up of four half V trough parabolic reflecting systems, presented in previous works by Stanislas Sanfo, and Abdoulaye Ouedraogo, 2015. The Fig.2 shows a profile view of that off axis Gregorian concentrators. It is characterized by a centered main axis that carried a symmetric plane, two distant half V trough parabolic primary reflectors, two stack V trough parabolic secondary reflectors with their axis inclined from β_{rs} regarding to the primary reflectors axis. The four V trough parabolic components have the same F-number so that:

$$f_{\#rp} = f_{\#rs} = f_{\#} = \frac{f_{rp}}{D_{rp}} \quad (9)$$

Where: f_{rp} is the foci distance of the primary reflector, D_{rp} is the primary reflector aperture.

The height of the receiver h_{rc} and the height of the secondary reflector h_{rs} according to the primary reflector are considered by dimensionless length quantities as follow:

$$\overline{h_{rc}} = \frac{h_{rc}}{f_{rp}} \quad (10)$$

$$\overline{h_{rs}} = \frac{h_{rs}}{f_{rp}} \quad (11)$$

On purpose $\overline{h_{rs}}$ operate a change on the level of the sun concentration ratio, while $\overline{h_{rc}} \rightarrow 1$ is a needed setup for the receiver in order to obtain a particular distribution of the irradiation that is uniformed over the receiver (Stanislas Sanfo, et al., 2015.). The chosen cases of study labeled Config1, Config2 and Config3 have parameters define as presented in Table 1. Their reflectors have the same curvature establishes by $f_{\#} = 0.6869$ and they differ from each other regarding to the level of the concentration ratio establishes by taking different values of $\overline{h_{rs}}$.

Table 1. Study cases optical configuration

Study cases	$\overline{h_{rs}}$	$\overline{h_{rc}}$	$f_{\#}$	β_{rs}
Config1	1.05	1	0.6869	25°
Config2	1.10	1	0.6869	25°
Config3	1.20	1	0.6869	25°

As stated in Table 2, we choose the PILLBOX model (Veynandt F., 2011., Cole I.R., et al. 2015.) for the representation of the sun in reference. The physical barrier lead by the components are taken into account for the two programs with a transparency parameter stated at 0 for the negative marked surfaces (refer to fig.1).

Table 2. Material setup according to the approach

Parts	***Program***	***Reflectivity***	***SigmaSlope***	***Distribution***
Reflector	Reference	1	0	PILLBOX
	MCMRDF-3D	1	0	PILLBOX similar
Receiver	Reference	0	0	PILLBOX
	MCMRDF-3D	0	0	PILLBOX similar

The simulations are carried out on the basis of an equivalent number of realizations. We use 931931 rays to carry out the first step of our new approach in MCMRDF-3D, we then transpose the pattern along 0.5 m in longitudinal direction with a step of 1 mm. On reference, to keep the same level of precision, we use 931931 x 501 rays. We point out that the reference software provides data which need to be post processed on the Mathematica software. For the calculation times we are going to consider the two software processing for providing final result.

MCMRDF-3D has been implemented on Matlab. The impact points of the rays on reflectors are determined using a more precise method. The interaction points between the rays and a reflected surface are determined in the local coordinate of the concerned reflector.

To analyze the flow map the relative positions over the receivers which showed to be dimensionless length quantities are carried for transversal direction and longitudinal direction respectively as follow:

$$l_X = \frac{X - X_{min}}{X_{max} - X_{min}} \quad (12)$$

$$l_Y = \frac{Y - Y_{min}}{Y_{max} - Y_{min}} \quad (13)$$

Then the three dimensional distribution $C(l_X\,;\,l_Y)$ is presented as the local number of sun or the proportional flux regard to the DNI at the local relative position $(l_X\,;\,l_Y)$ by:

$$C(l_X\,;\,l_Y) = \frac{\psi(R(l_X\,;\,l_Y))}{DNI} \quad (14)$$

III. RESULTS AND DISCUSSIONS

In this section, we compare the processing and results data from reference software and MCMRDF-3D for the determination of the distribution of the irradiation provided by studied cases Config1, Config2 and Config3.

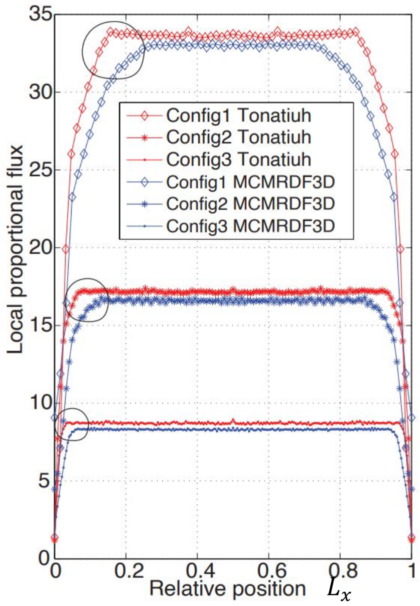


Fig.3. *Irradiation distribution plot in transverse direction according to the program: REFERENCE in red color and MCMRDF-3D in blue color, regarding to the concentrators setup*

Fig.3 show the projections of all the irradiation in the transverse plane. It points out the differences more clearly. In this figure, we notice that the curves have relatively the same progressions according to the non-dimensional position, but do not overlap perfectly.

Taking into account the symmetry of the curves, the differences are noticeable around the borders mainly we marked by circles on the picture. We can notice that more the height $\overline{h_{rs}}$ is short, more the difference is marked. We assume that these differences can be due to the accuracy of the method used to determine the interaction points in our program compare to Reference Software. Typically, optical analysis software uses a least-mean-square method together with linear or spline interpolation of the surface in multiple discrete small area (Sergio Ortiz, et al., 2009.). In our works we used a more precise and efficient way of finding the intersection points. The local axis of each V-trough parabolic optical surface is determined using a method described in previous works (Stanislas Sanfo, et al., 2015.) and the intersection points are found by resolving the system of equations in local coordinates before changing to global coordinates.

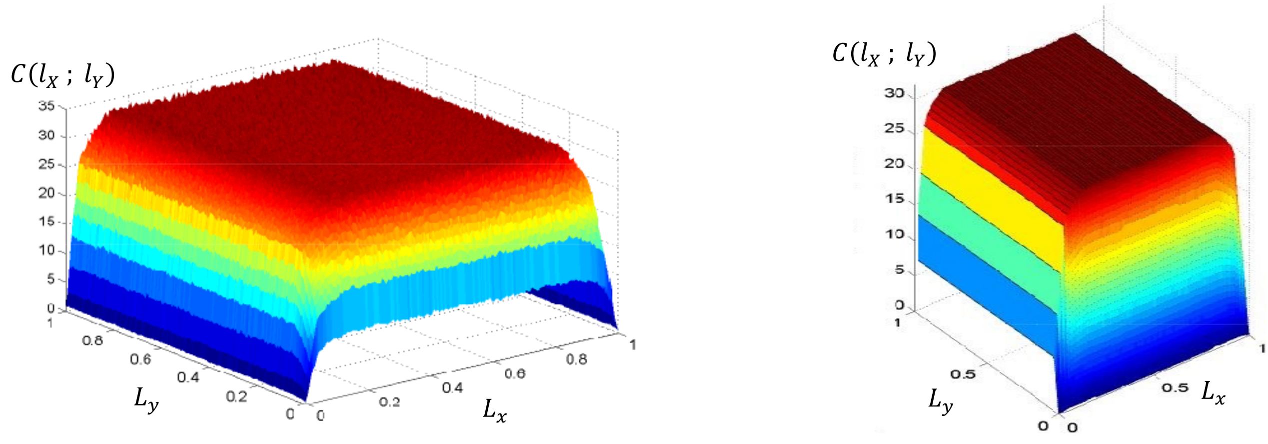
Despite these differences, the comparison of the two sets of results places the approach incorporated in the code MCMRDF-3D an interesting manner option to describe the irradiation profile gave by a concentrating optics system over its receiver.

Table 3. Results of comparison between MCMRDF-3D and the reference Software

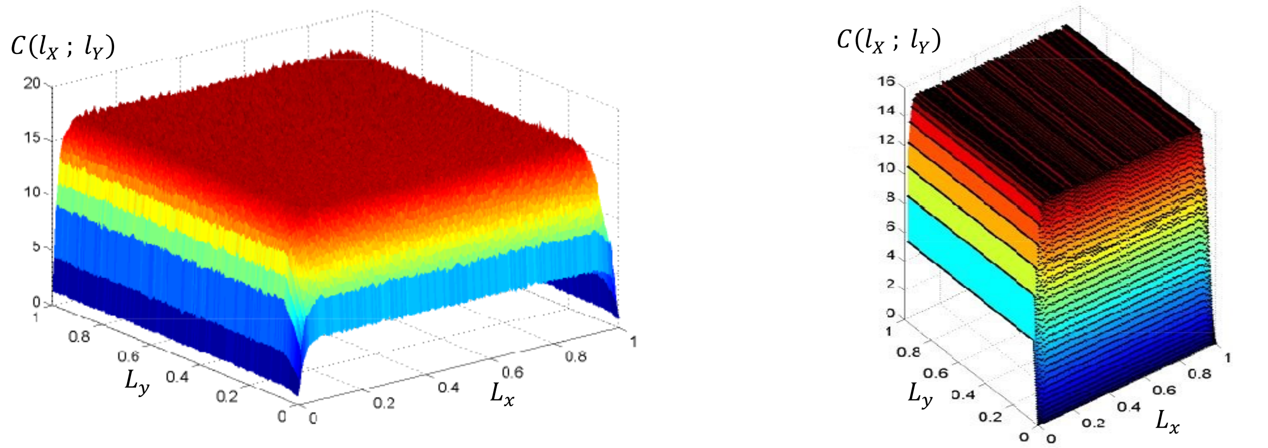
Calculation time (seconds)		***Storage data size (Mo)***	
Reference	MCMRDF-3D	Reference	MCMRDF-3D
3,833	32	13,590	42
4,541	32	13,550	43
3,333	32	11,630	44

Table 3 presents the computation process time and data results for the two described programs applied to determine the energy harvested by the receiver of the three stated concentrators. The results carried out on the basic configurations Config1 Config2 and Config3 show tremendous gap between the two programs. Our program is much faster than the reference software. Indeed, the times of simulation obtained for these tests are approximately 320 times lower with MCMRDF-3D than the reference combined to its post processing software. In fact, running MCMRDF-3D program spent computing time of 32 seconds at each simulation comparatively to reference software which spent over 3,801 to 4,509 seconds. MCMRDF-3D required 42Mo of hardware memory comparatively to over 11.63 to 13.59 Go required by the reference. This shows that the MCMRDF-3D program leads to very advantageous savings in computation time and physical memory.

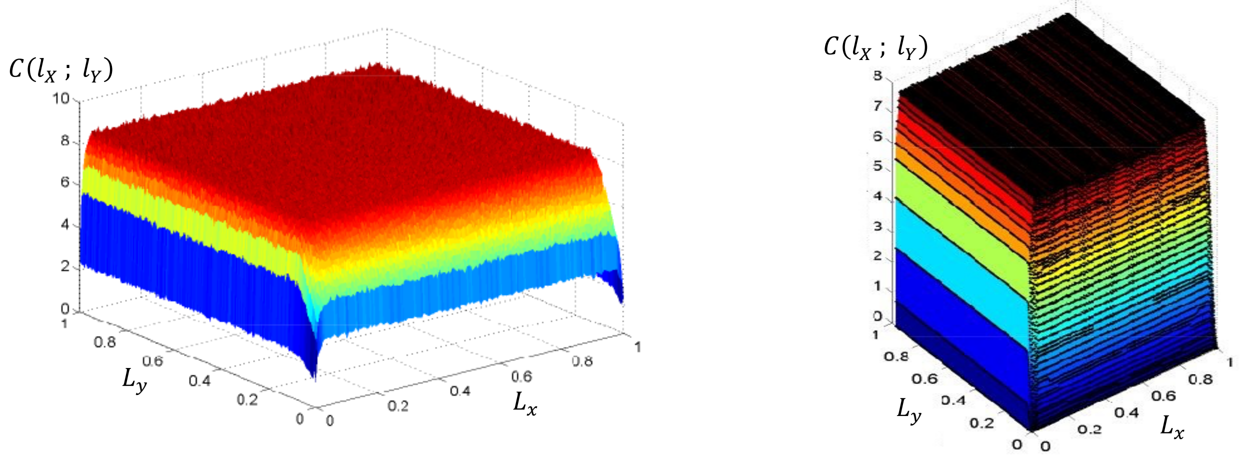
It should be emphasized that these advantages are due to the singularity of the simulation problems addressed; which allowed us to build a large-scale flow diagrams using a small number of practical realizations. Conversely, the Reference is designed to cover broader and more extensive fields of applications. Fig.4 presents the isodiagram map flow of the irradiation over the receiver surface. As it can be seen from Fig.4 a, b, c the obtained flow maps from MCMRDF-3D program at the right are consistent with the flow maps obtained from Reference presented at the left.



(a)



(b)



(c)

Fig.4. *Flow map of the irradiation over the receivers according to the program: Reference at the left and MCMRDF-3D at the right, and according to the concentrator optical setup a) Config1 , b) Config 2 and c) Config 3*

For a giving concentrator the both show the same tendency of $C(l_X\,;\,l_Y)$ in reaching a certain distribution level. However it appears a difference between the local proportional flux $C(l_X\,;\,l_Y)$ from Reference which is slightly greather than those of MCMRDF-3D

IV. CONCLUSION

In this paper, we presented a new approach incorporated in the program MCMRDF-3D with the aim of minimizing the computation time and storage memory for the determination of the distribution of irradiation. The MCMRDF-3D program has been successfully compared to reference software. The flow map of irradiation show a significant similarity for the three studied concentrators. The results showed a significant economy of computation time and memory storage compared to Reference software. It performs a quick calculation with an acceptable confidence interval related to geometry calculation errors. For similar level of results, it took only 32.021 seconds for the MCMRDF-3D program to make a simulation whereas the Reference software took more than 100 times longer time. Besides, it required 320 times more of storage capacities for Reference Software than MCMRDF-3D program.

Then we can concluded that the challenge of saving time and computer capacity has been for the specific problem of symmetric solar optical systems is well done. The new approach has a considerable lower number of realizations and gives good results, with an acceptable margin of error. These results are promising future issue for considering real behavior, like non specular properties of optical components, by mean of few starting rays from the sun and numerous number of realizations for each ray interaction with matter.

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